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A new simple tiling, with unusual properties, by a polyhedron with 14 faces

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A monotypic simple tiling by a 14-face polyhedron that does not admit an isohedral tiling is described. The tiling is triclinic and contains four distinct, but combinatorially equivalent, kinds of tile.

A polyhedron has a graph that is planar and three-connected (*i.e.* at least three vertices and their incident edges have to be deleted to separate the graph into two disjoint parts). In a *simple polyhedron*, two faces meet at each edge and three at each vertex. In a *simple tiling* of three-dimensional Euclidean space, the tiles are simple polyhedra and two meet at each face, three at each edge and four at each vertex. An *isohedral* tiling is one in which any two tiles are related by an isometry of the symmetry group of the tiling. A *monotypic* tiling is one in which all tiles are combinatorially equivalent (have the same graph). An isohedral tiling is monotypic but not necessarily *vice versa*.

Simple tilings are of considerable interest as idealized models of foams and other physical systems (Sadoc & Rivier, 1999), their nets (the skeleton of vertices and edges) are of interest in crystal chemistry as the framework types of real and hypothetical zeolites (Delgado-Friedrichs *et al.*, 1999), and they present a number of interesting problems. The most celebrated of these is the Kelvin problem (Weaire, 1996), which asks for the lowest-energy (smallest surface area) tiling for tiles of a given volume. Among other things, this has prompted numerous studies of isohedral simple tilings. A recent study is that of Delgado-Friedrichs & O'Keeffe (2005) who showed that: (*a*) there are no isohedral simple tilings by tiles with less than 14 faces; (*b*) all 14-face tiles of isohedral simple tilings have only faces of 4, 5 or 6 sides (4-6 polyhedra); (*c*) of the 59 different 4-6 polyhedra with 14 faces there are 10 different isohedral tilers that produce 23 distinct isohedral tilings. These results have been confirmed by Komarov *et al.* (2007), who also give a full account of earlier work.

The question of whether a given combinatorial type of polyhedron admits monotypic tilings has also attracted considerable attention. It is known (Schulte, 1985) that there are *non-tilers*, isomorphic copies of which will not tile space in a locally finite and face-to-face fashion; the cuboctahedron is an example. On the other hand, the dual of a k vertex-transitive simple tiling is a k tile-transitive tiling by tetrahedra and necessarily monotypic.[1] In fact, simplicial polyhedra (those with only triangular faces) in general are tilers (Grünbaum *et al.*, 1984).

In this report, we describe some properties of a new monotypic simple tiling discovered by one of us (RG). The tile in this structure again has 14 faces but is distinct from the 10 isohedral tilers and is the unique 14-face 4-6 simple polyhedron with one quadrilateral face. It has symmetry m; its Schlegel diagram is shown in Fig. 1. The combinatorial symmetry of the net of this tiling is $P\bar{1}$ as determined by the program *Systre* [the method is described by Delgado-Friedrichs & O'Keeffe (2003)]. The unit cell contains eight tiles, each of symmetry 1, that are four pairs of enantiomers. The inversion centers are located in 4- and 6-sided faces. An illustration of a repeat unit made with the program *3dt* is shown in Fig. 2. The structure has 24 kinds of vertex, 48 kinds of edge, 32 kinds of face and 4 kinds of tile (transitivity 24 48 32 4).

As far as we know, this is the first example of a monotypic simple tiling by a polyhedron that does not admit an isohedral tiling and it raises some interesting questions. What polyhedra admit monotypic simple tilings other than the known isohedral tilers? In particular, do any of the other 14-face 4-6 polyhedra admit monotypic tilings? Do polyhedra with less than 14 faces admit a monotypic simple tiling? It

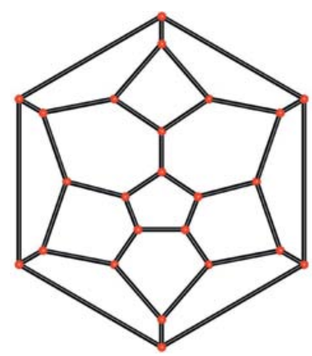

Figure 1
The Schlegel diagram of the polyhedron.

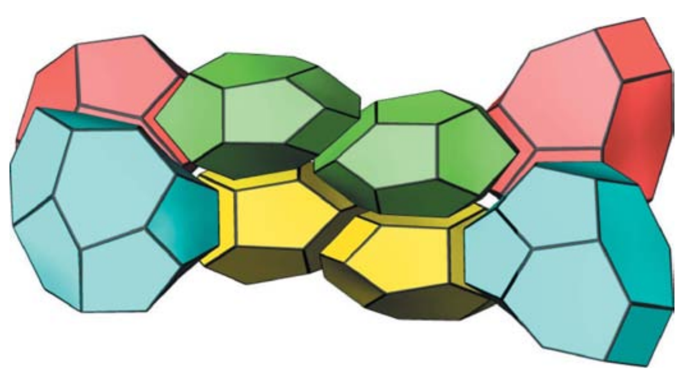

Figure 2
A repeat unit of the tiling. Tiles of the same color are related by inversion.

[1] By k tile (vertex) transitive, we mean that there are k kinds of tile (vertex) in which all tiles (vertices) of one kind are related by symmetry but there is no symmetry operation relating tiles (vertices) of different kinds.

is known only that the average face size in a simple tiling must be <6 and ≥9/2 (Luo & Stong, 1993) so the average number of faces per tile is ≥8. For an example of a simple tiling with average face size approaching that lower limit, see O'Keeffe (in Sadoc & Rivier, 1999).

We remark also that intrinsically triclinic structures rarely arise in such studies. For example, of the many hundreds of known 3-periodic packings of one kind of sphere, there is exactly one that is triclinic (Fischer & Koch, 2002). The net of this structure is the only triclinic entry in the RCSR database (http://rcsr.anu.edu.au) of over 1000 nets, and there are no triclinic examples among the thousands of nets in the EPINET database (http://epinet.anu.edu.au).

Crystallographic data for an embedding with edge lengths all equal to 1 are $a = 5.770$, $b = 5.806$, $c = 16.834$, $\alpha = 94.81$ $\beta = 94.39$, $\gamma = 90.62^\circ$. The centroids of the polyhedra are at ±(0.4110, 0.0920, 0.8775; 0.0325, 0.5904, 0.8782; 0.4086, 0.4674, 0.6215; 0.0899, 0.9099, 0.3794). The coordinates of the vertices of the net of the structure are being entered in the RCSR database with the net symbol **rug**.

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